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* Tortional Rigidity Of Hollow Cylinder is greater than the tortional rigidity of Solid Cylinder:—

Hollow Cylinder:— It is interesting to note that a hollow shaft has greater tortional rigidity than a solid shaft of the same material, mass and length.

Let the couple on a hollow cylinder of length L, external and internal radius r_2 and r_1 respectively is given by

$$T = \frac{2\pi\eta\theta}{l}\int_{r_1}^{r_2} x^3 \cdot dx$$

$$= \frac{2\pi\eta\theta}{l}\left[\frac{x^4}{4}\right]_{r_1}^{r_2}$$

$$= \frac{2\pi\eta\theta}{l}\left[\frac{r_2^4 - r_1^4}{4}\right]$$

$$\therefore C' \text{ (tortional rigidity)} = \frac{\eta\pi(r_2^4 - r_1^4)}{2l}$$

Similarly,

C (tortional rigidity of a solid cylinder)

$$= \frac{\eta\pi r^4}{2l}$$

$$\therefore \frac{C'}{C_1} = \frac{\eta\pi(r_2^4 - r_1^4)}{2l} \Big/ \frac{\eta\pi r^4}{2l}$$

$$= \frac{\eta\pi(r_2^4 - r_1^4)}{2l} \times \frac{2l}{\eta\pi r^4}$$

$$\therefore \frac{C'}{C} = \frac{r_2^4 - r_1^4}{r^4}$$

$$= \frac{(r_2^2 - r_1^2)(r_2^2 + r_1^2)}{r^4} \quad -(i)$$

Since masses of cylinder are equal.

$$\pi (r_2^2 - r_1^2) l \rho = \pi r^2 l \rho$$

or, $r_2^2 - r_1^2 = r^2$

$\therefore$ from eqn (i), We get,

$$\frac{C'}{C} = \frac{r^2 (r_2^2 - r_1^2)}{r^4}$$

$$= \frac{r_2^2 + r_1^2}{r^2} \quad \because r_1^2 = -r_1^2 + 2r_1^2$$

$$\therefore \frac{C'}{C} = \frac{r_2^2 - r_1^2 + 2r_1^2}{r^2} \quad \because r_2^2 - r_1^2 = r^2$$

$$\therefore \frac{C'}{C} = \frac{r^2 + 2r_1^2}{r^2}$$

$$= 1 + \frac{2r_1^2}{r^2}$$

$\because 1 + \frac{2r_1^2}{r^2}$ will be greater than 1.

i.e., $\frac{C'}{C} > 1$

$\therefore C' > C$. So tortional rigidity of hollow cylinder is greater than torsional rigidity of solid cylinder.